

Magnetic strings in Lattice QCD as Nonabelian Vortices

A.GORSKY^a AND V.ZAKHAROV^{a,b}

a. Institute of Theoretical and Experimental Physics

B. Cheremushkinskaya ul. 25, 117259 Moscow, Russia

b. INFN - Sezione di Pisa, Largo Pontecorvo, 3, 56127 Pisa, Italy

Abstract

Lattice studies indicate existence of magnetic strings in QCD vacuum. We argue that recently found nonabelian strings with rich worldsheet dynamics provide a proper pattern for the strings observed on the lattice. In particular, within this pattern we explain the localization of the monopole-antimonopole pairs on the magnetic string worldsheet and the negative contribution of the magnetic strings into the vacuum energy and gluon condensate. We suggest the D2 brane realization of the magnetic string which explains the temperature dependence of its tension.

1 Introduction

Explanation of the QCD vacuum structure remains a challenging problem. Recently some progress has been made in the lattice studies and their interpretation [1]. In particular the essential contribution from the 2d surfaces (strings) and 3d volumes (domain walls) with some unusual properties to the vacuum characteristics has been found. For our purposes, the key properties of the magnetic strings observed on the lattice can be summarized as follows (for references see Section 4 below and reviews [1]):

- The tension of the magnetic string vanishes below the critical temperature and they percolate through the vacuum, forming a kind of a vacuum condensate
- The worldsheet of magnetic string is populated by monopole-antimonopole pairs
- Above the temperature of the deconfinement phase transition magnetic string becomes tensionful

Most recently, it was argued that

- The magnetic strings become a component of Yang-Mills plasma [2]

and the first measurements indicate surprisingly that

- The contribution of the strings to the gluon condensate and 4d bulk vacuum energy is opposite in sign compared to its total value [3]

A natural question concerns the very existence of strings with such properties in the continuum theory. The goal of this note is to argue that nonabelian magnetic strings found recently in the SUSY gauge theories naturally provide the desired pattern. We are not aiming to prove rigorously that nonabelian strings populate QCD vacuum. However our considerations clearly indicate that this kind of object fits perfectly the lattice data.

The nonabelian strings which are essentially twisted Z_N strings with orientational moduli have been first found in SUSY context [4, 5]. However, later it was recognized that they do exist in non SUSY theories as well [6] (see [7, 8, 9] for reviews). The key property of the nonabelian strings which distinguishes them from the other objects discussed in this context is highly nontrivial worldsheet theory which in the simplest examples can be identified with $CP(N-1)$ sigma model. Moreover it was found that kinks on the worldsheet are nothing else but the 4d monopoles "trapped" by the string [10, 11]. In nonsupersymmetric case $CP(N-1)$ worldsheet theory is in the confinement phase [13] so that only kink-antikink pairs exist which parallels the lattice QCD observations. It was also argued recently that nonabelian strings could play an essential role in the Seiberg duality [14, 15].

As is mentioned above, the very recent lattice data indicates that magnetic strings contribute to the vacuum energy and gluon condensate with the unexpected sign [3]. On the other hand, it was found long time ago [12] that vacuum energies in 4d gauge theories and 2d $CP(N-1)$ sigma model have opposite signs. We argue that this old observation provides a pattern for an interpretation of the lattice data [3].

The lattice data suggests that the tension of the magnetic string is zero below the deconfinement temperature T_c and the question is whether the nonabelian strings share this property. To get insight into the problem we will use the brane realization of the nonabelian string as D2 brane in the particular supergravity background. Within this picture we argue that interpretation of the magnetic string as the wrapped D2 brane explains the temperature dependence of the tension. The crucial point is that the worldsheet action on the magnetic string consists of two parts: "space" part presumably involving the Nambu-Goto type contribution as well as "internal" $CP(N-1)$ part responsible for the rich structure of the worldsheet theory. The vanishing of the "space" tension of the string below the critical temperature does not imply the vanishing of "internal" part which is still responsible for the nontrivial dynamics at small temperature. The change of the background above the critical temperature results in an interesting phenomenon that the properties of the time- and space-oriented magnetic strings become different.

The paper is organized as follows. In Section 2 we explain the construction of the nonabelian string solution in the simple model. In Section 3 we argue that the nonabelian string pattern explains the negative contributions to the vacuum energy and the gluon condensate and temperature dependence of the magnetic string tension. In Section 4 we briefly compare our picture with the available lattice data on the magnetic strings. Section 5 involves the brief discussion on the results obtained and some unsolved issues.

2 Nonabelian strings

Here we review the simplest model which can be used to analyze nonabelian strings. The gauge group of the model is $SU(N) \times U(1)$. Besides $SU(N)$ and $U(1)$ gauge bosons the model contains N scalar fields charged with respect to $U(1)$ which form N fundamental representations of $SU(N)$. It is convenient to write these fields as $N \times N$ matrix $\Phi = \{\varphi^{kA}\}$ where k is the $SU(N)$ gauge index while A is the flavor index,

$$\Phi = \begin{pmatrix} \varphi^{11} & \varphi^{12} & \dots & \varphi^{1N} \\ \varphi^{21} & \varphi^{22} & \dots & \varphi^{2N} \\ \dots & \dots & \dots & \dots \\ \varphi^{N1} & \varphi^{N2} & \dots & \varphi^{NN} \end{pmatrix}. \quad (1)$$

The action of the model has the form

$$\begin{aligned} S = & \int d^4x \left\{ \frac{1}{4g_2^2} (F_{\mu\nu}^a)^2 + \frac{1}{4g_1^2} (F_{\mu\nu})^2 \right. \\ & + \text{Tr} (\nabla_\mu \Phi)^\dagger (\nabla^\mu \Phi) + \frac{g_2^2}{2} [\text{Tr} (\Phi^\dagger T^a \Phi)]^2 + \frac{g_1^2}{8} [\text{Tr} (\Phi^\dagger \Phi) - N\xi]^2 \\ & \left. + \frac{i\theta}{32\pi^2} F_{\mu\nu}^a \tilde{F}^{a\mu\nu} \right\}, \quad (2) \end{aligned}$$

where T^a stands for the generator of the gauge $SU(N)$,

$$\nabla_\mu \Phi \equiv \left(\partial_\mu - \frac{i}{\sqrt{2N}} A_\mu - i A_\mu^a T^a \right) \Phi, \quad (3)$$

and θ is the vacuum angle. The action (2) represents a truncated bosonic sector of the $N=2$ SUSY model. The last term in the second line forces Φ to develop a vacuum expectation value (VEV) while the last but one term forces the VEV to be diagonal,

$$\Phi_{\text{vac}} = \sqrt{\xi} \text{diag} \{1, 1, \dots, 1\}. \quad (4)$$

We assume that the parameter ξ to be large,

$$\sqrt{\xi} \gg \Lambda_4, \quad (5)$$

where Λ_4 is the scale of the four-dimensional theory (2). That is we are in the weak coupling regime as both couplings g_1^2 and g_2^2 are frozen at a large scale.

The vacuum field (4) results in the spontaneous breaking of both gauge and flavor $SU(N)$'s. A diagonal global $SU(N)$ survives

$$SU(N)_{\text{gauge}} \times SU(N)_{\text{flavor}} \rightarrow SU(N)_{\text{diag}}. \quad (6)$$

yielding color-flavor locking in the vacuum.

To describe the topological argument providing the stability of the string one can combine the Z_N center of $SU(N)$ with the elements $\exp(2\pi i k/N) \in U(1)$ to get a topologically stable string solution possessing both windings, in $SU(N)$ and $U(1)$. In other words,

$$\pi_1(SU(N) \times U(1)/Z_N) \neq 0. \quad (7)$$

and this nontrivial topology amounts to winding of just one element of Φ_{vac} , for instance,

$$\Phi_{\text{string}} = \sqrt{\xi} \text{diag}(1, 1, \dots, e^{i\alpha(x)}), \quad x \rightarrow \infty. \quad (8)$$

These strings can be called elementary Z_N strings; their tension is $1/N$ -th of that of the ANO string. The ANO string can be viewed as a bound state of N Z_N strings.

The Z_N string solution can be written as follows [5]:

$$\begin{aligned} \Phi &= \begin{pmatrix} \phi(r) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \phi(r) & 0 \\ 0 & 0 & \dots & e^{i\alpha} \phi_N(r) \end{pmatrix}, \\ A_i^{\text{SU}(N)} &= \frac{1}{N} \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} (\partial_i \alpha) [-1 + f_{NA}(r)], \\ A_i^{\text{U}(1)} &= \frac{1}{N} (\partial_i \alpha) [1 - f(r)], \quad A_0^{\text{U}(1)} = A_0^{\text{SU}(N)} = 0, \end{aligned} \quad (9)$$

where $i = 1, 2$ labels coordinates in the plane orthogonal to the string axis and r and α are the polar coordinates in this plane. The profile functions $\phi(r)$ and $\phi_N(r)$ determine the profiles of the scalar fields, while $f_{NA}(r)$ and $f(r)$ determine the $SU(N)$ and $U(1)$ fields of the string solutions, respectively. These functions satisfy the following boundary conditions:

$$\begin{aligned}\phi_N(0) &= 0, \\ f_{NA}(0) &= 1, \quad f(0) = 1,\end{aligned}\tag{10}$$

at $r = 0$, and

$$\begin{aligned}\phi_N(\infty) &= \sqrt{\xi}, \quad \phi(\infty) = \sqrt{\xi}, \\ f_{NA}(\infty) &= 0, \quad f(\infty) = 0\end{aligned}\tag{11}$$

at $r = \infty$. These profile functions satisfy the first-order differential equations, namely,

$$\begin{aligned}r \frac{d}{dr} \phi_N(r) - \frac{1}{N} (f(r) + (N-1)f_{NA}(r)) \phi_N(r) &= 0, \\ r \frac{d}{dr} \phi(r) - \frac{1}{N} (f(r) - f_{NA}(r)) \phi(r) &= 0, \\ -\frac{1}{r} \frac{d}{dr} f(r) + \frac{g_1^2 N}{4} [(N-1)\phi(r)^2 + \phi_N(r)^2 - N\xi] &= 0, \\ -\frac{1}{r} \frac{d}{dr} f_{NA}(r) + \frac{g_2^2}{2} [\phi_N(r)^2 - \phi(r)^2] &= 0.\end{aligned}\tag{12}$$

The tension of this elementary string is

$$T_1 = 2\pi \xi.\tag{13}$$

while the tension of the ANO string is

$$T_{\text{ANO}} = 2\pi N \xi\tag{14}$$

which confirms its composite nature.

The elementary strings are essentially non-Abelian since besides trivial translational moduli, they give rise to moduli corresponding to spontaneous breaking of a non-Abelian symmetry. Indeed, while the “flat” vacuum is $SU(N)_{\text{diag}}$ symmetric, the solution (9) breaks this symmetry down. This means that the worldsheet theory of the elementary string moduli is the $CP(N-1)$ sigma model.

To obtain the non-Abelian string solution from the Z_N string (9) we apply the diagonal color-flavor rotation preserving the vacuum (4). It is useful to pass to the singular gauge where the scalar fields have no winding at infinity, while the string flux comes from the

vicinity of the origin. In singular gauge we have

$$\begin{aligned}\Phi &= U \begin{pmatrix} \phi(r) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \phi(r) & 0 \\ 0 & 0 & \dots & \phi_N(r) \end{pmatrix} U^{-1}, \\ A_i^{\text{SU}(N)} &= \frac{1}{N} U \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} U^{-1} (\partial_i \alpha) f_{NA}(r), \\ A_i^{\text{U}(1)} &= -\frac{1}{N} (\partial_i \alpha) f(r), \quad A_0^{\text{U}(1)} = A_0^{\text{SU}(N)} = 0, \end{aligned} \tag{15}$$

where U is a matrix $\in \text{SU}(N)$. This matrix parameterizes orientational zero modes of the string associated with flux embedding into $\text{SU}(N)$. The orientational moduli encoded in the matrix U were first observed in [4, 5].

Let us discuss the worldsheet description of the nonabelian string. It is important that there are two independent contribution from "space" and "internal" terms. The space-time action does not reduce purely to the Nambu-Goto term which is only the first approximation term. The corresponding tension is proportional to ξ . To obtain the kinetic term in the "internal" action we follow the standard logic in the derivation of the low-energy action in the moduli approximation. That is we substitute our solution, which depends on the moduli n^l , in the action, assuming that the fields acquire a dependence on the coordinates x_k via $n^l(x_k)$. Then we arrive at the $CP(N-1)$ sigma model (for details see [8]),

$$S_{CP(N-1)}^{(1+1)} = 2f \int dt dz \left\{ (\partial_k n^* \partial_k n) + (n^* \partial_k n)^2 \right\}, \tag{16}$$

where the coupling constant f is given by a normalizing integral defined in terms of the string profile functions which yields

$$f = \frac{2\pi}{g_2^2}. \tag{17}$$

that is two-dimensional coupling constant is determined by the four-dimensional non-Abelian coupling.

The relation between the four-dimensional and two-dimensional coupling constants (17) is obtained at the classical level. In quantum theory both couplings run hence we have to specify a scale at which the relation (17) takes place. The two-dimensional $CP(N-1)$ model is an effective low-energy theory good for the description of internal string dynamics at low energies, much lower than the inverse thickness of the string which, in turn, is given by $g_2\sqrt{\xi}$. Therefore, $g_2\sqrt{\xi}$ plays the role of a physical ultraviolet cutoff in (16). Below this scale, the coupling f runs according to its two-dimensional renormalization-group flow.

The sigma model (16) is asymptotically free hence at large distances it gets into the strong coupling regime. The running coupling constant as a function of the energy scale E at one loop is given by

$$4\pi f = N \ln \left(\frac{E}{\Lambda_{CP(N-1)}} \right) + \dots, \quad (18)$$

where $\Lambda_{CP(N-1)}$ is the dynamical scale of the $CP(N-1)$ model. As was mentioned above, the UV cut-off of the sigma model at hand is determined by $g_2\sqrt{\xi}$. Hence,

$$\Lambda_{CP(N-1)}^N = g_2^N \xi^{N/2} e^{-\frac{8\pi^2}{g_2^2}}. \quad (19)$$

In the bulk theory, due to the VEV's of the scalar fields, the coupling constant is frozen at $g_2\sqrt{\xi}$. There are no logarithms in the bulk theory below this scale and the logarithms of the world-sheet theory take over.

The brane realization of the nonabelian strings can be captured from the brane realization of the $CP(N-1)$ models. It corresponds to the theory on the worldvolume of D2 brane in the background of two NS5 branes and N D4 branes. The brane geometry can be seen from the D2 worldvolume perspective or N D4 brane worldvolume perspective providing the rationale for the relation between the physics of 2d $CP(N-1)$ model and 4d SQCD [18].

3 Magnetic strings versus nonabelian strings

3.1 Monopole pairs on the worldsheet

Let us show that the pattern of the nonabelian strings provides the explanation of the properties of the magnetic strings observed on the lattice. The first point we would like to note is that from the discussion above it is clear that monopole pairs are present on the nonabelian string indeed.

The worldsheet theory is nonsupersymmetric σ -model which has single vacuum state and the spectrum consists of kink-antikink bound states [13]. These bound states can be identified with monopole-antimonopole bound states from the four-dimensional viewpoint. The IR scale Λ_{CP} is generated in the worldsheet theory and can be related to the scale in the bulk theory. The masses of the bound states in the theory are of order Λ_{CP} and they cannot be found exactly since worldsheet theory is in strong coupling regime. In the SUSY setup one can consider massive flavors yielding the quasiclassical picture of the bound states. In the nonSUSY case we have no such simple argumentation. Note however that the monopoles in the Higgs phase on the string worldsheet are smoothly related to the T'Hooft-Polyakov monopoles via the continuous deformation in the parameter space (see a recent discussion in [20]).

If one introduces θ term in the bulk theory then due to Witten effect monopoles acquires the electric charge and becomes a dyon. Similar picture happens on the worldsheet as well. The θ term penetrates into the worldsheet theory and kink in the worldsheet theory acquires the global charge.

3.2 Vacuum energy and gluon condensate

In view of the recent lattice measurements [3] of contribution of magnetic strings into Yang-Mills plasma energy we will consider the energy issue in the context of the nonabelian strings. There are two contributions to the energy associated with dynamics in space-time and internal space, respectively. These contributions can be treated separately. Our basic observation is that the contribution from the internal, $CP(N-1)$ part is in fact negative and opposite in sign to its total value.

First, note that vacuum energy (at vanishing temperature) in the Yang-Mills theory is related to the conformal anomaly:

$$E_{vac}^{YM} = \frac{1}{4} \langle 0 | \theta_{\mu\mu}^{YM} | 0 \rangle = \langle 0 | -\frac{b_0 \alpha_S}{32\pi} Tr G^2 | 0 \rangle \quad (20)$$

where b_0 is the beta function coefficient. Similar relation holds in $CP(N-1)$ model as well. Namely

$$E_{vac}^{CP} = \frac{1}{2} \langle 0 | \theta_{\mu\mu}^{CP} | 0 \rangle = \frac{N}{8\pi} \Lambda_{CP}^2 \quad (21)$$

The gluon condensate $\langle Tr G^2 \rangle$ gets contribution from the nonabelian strings since the internal tension is proportional to inverse gauge coupling

$$\langle Tr G^2 \rangle_{tot} \propto \frac{d}{d(1/g^2)} \log Z \propto \langle Tr G^2 \rangle_{YM} + C_{CP} \langle Tr G^2 \rangle_{CP} \quad (22)$$

The two-dimensional contribution from the nonabelian strings comes from the vacuum expectation value of two-dimensional conformal anomaly in $CP(N-1)$ model which has the opposite sign [12] compared to the total value

$$\langle Tr G^2 \rangle_{CP} \propto \langle 0 | \theta_{\mu\mu}^{CP} | 0 \rangle = \frac{N}{8\pi} \Lambda_{CP}^2 \quad (23)$$

The value of the dimensionful constant C_{CP} is determined by the density of the strings and we can not estimate its value at a moment.

Let us emphasize that we considered only the nonperturbative contributions to the vacuum energy which is determined by the nonperturbative contribution to the conformal anomaly. The contribution to the vacuum energy from the space part of the string action vanishes since it is proportional to the string tension.

3.3 Low-energy Theorems and Dilaton

Condensation of the magnetic string, observed on the lattice, requires the consideration of the back reaction of the single nonabelian string on the bulk fields. Below we use the low-energy theorems in $CP(N-1)$ model to argue that scalar mode on the worldsheet contributes negatively to the mass squared of the corresponding mode in four dimensions contrary to the pseudoscalar case.

In the bulk theory, for any operator A there holds the dilatation Ward identity:

$$i \int d^4x \langle 0 | \theta_{\mu\mu}^{YM}(x) A(0) | 0 \rangle = -d_A \langle 0 | A | 0 \rangle \quad (24)$$

where d_A is the canonical dimension of the operator A . This equation follows from the very fact of asymptotic freedom in the theory. Similar arguments apply to the worldsheet theory and the corresponding dilatation Ward identity reads [12]:

$$i \int d^2x < 0 | \theta_{\mu\mu}^{CP}(x), A(0) | 0 > = -d_A < 0 | A | 0 > \quad (25)$$

where we consider the correlator of the $\theta_{\mu\mu}$ with the arbitrary operator in the $CP(N-1)$ sigma model.

Some of the operators A are of the special interest. Consider first the operator $A = TrG^2$ so that the corresponding low-energy theorem reads as:

$$i \int d^4x < 0 | TrG^2(x), TrG^2(0) | 0 > = S_{YM}(0) \propto < 0 | TrG^2 | 0 > \quad (26)$$

In YM theory the r.h.s. is positive that is if we consider the particle saturating $S(0)$ it has the positive mass. This particle naturally could be related to the dilaton ϕ because of the standard coupling of the dilaton $e^\phi TrG^2$. On the other hand it is clear from the arguments above that this correlator has contribution from the string of the form

$$i \int d^2x < 0 | \theta_{\mu\mu}^{CP}(x), \theta_{\mu\mu}^{CP}(0) | 0 > = S_{CP}(0) \quad (27)$$

The low-energy theorem yields $S_{CP} < 0$ [12] which corresponds to tachyonic contribution to the particle in the intermediate state. The total mass of scalar is positive while the stringy contribution is negative.

Let us compare the bulk-worldsheet interplay of the dilaton dynamics with the similar consideration concerning axion [16]. It was shown in [16] that two-dimensional axion due to the mixing with photon is responsible for deconfinement on the worldsheet. The reason is that because of this mixing worldsheet photon becomes massive and linear confinement disappears. On the other hand the nonabelian string does not cause strong modification of the bulk dynamics and results only on the axion emission halo around the string.

One can consider the correlator of the topological charge densities

$$\frac{d^2 \log Z}{d^2 \theta} = \int d^4x < 0 | TrG\tilde{G}(x), TrG\tilde{G}(0) | 0 > = P_{YM}(0) \quad (28)$$

which can be saturated by the axion in the intermediate state (we have no quarks). Since the four dimensional θ term penetrates into the worldsheet theory the $P(0)$ has the two-dimensional contribution

$$i \int d^2x < 0 | F(x)F(0) | 0 > = P_{CP}(0) \quad (29)$$

where $F = \epsilon_{\mu\nu} \partial_\mu A_\nu$ and A_μ is the auxiliary abelian gauge field in $CP(N-1)$ model which acquires the mass at the quantum level. The contribution of the P_{CP} to the total $P(0)$ depends on the density of the nonabelian strings but the crucial point now is the sign of this contribution. Namely, it is known [12] that the value of $P_{CP}(0)$ is positive and has the same sign as the total correlator. That is, we have opposite influence of the nonabelian strings on the dynamics of the dilaton and axion. The string tends to decrease the mass of scalar and acts oppositely in the pseudoscalar case.

3.4 Magnetic string tension from brane perspective

Let us discuss the brane realization of the magnetic string. To begin with, consider the weak-coupling nonabelian string. In the $N = 2$ SQCD case nonabelian string is perfectly identified as D2 brane stretched between two NS5 branes displaced at the large distance ξ is some direction. According to the standard logic the tension of the nonabelian string turns out to be proportional to ξ that is quasiclassical analysis is reasonable.

The geometry of the nonSUSY QCD is not established well enough. However, the natural starting point is the geometry provided by the set of D4 branes wrapped around one compact dimensions [17]. We shall consider the pure gauge sector and does not discuss chiral matter in this note.

We shall assume the large N_C limit and consider the supergravity approximation. In this approximation the geometry looks as $M_{10} = R_{3,1} \times D \times S^4$ and the corresponding metric reads as

$$ds^2 = \left(\frac{u}{R_0}\right)^{3/2}(-dt^2 + \delta_{ij}dx^i dx^j + f(u)dx_4^2) + \left(\frac{u}{R_0}\right)^{-3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

$$e^\Phi = \left(\frac{u}{R_0}\right)^4, \quad F_4 = \frac{3N_c \epsilon_4}{4\pi}, \quad f(u) = 1 - \left(\frac{u_\Lambda}{u}\right)^3 \quad (30)$$

where $R_0 = (\pi g_s N_c)^{1/3}$ and $R = \frac{4\pi}{3}\left(\frac{R_0^3}{u_\Lambda}\right)^{1/2}$. The coupling constant of Yang-Mills theory is related to the radius of the compact dimension R as follows

$$g_{YM}^2 = \frac{8\pi^2 g_s l_s}{R}$$

At zero temperature theory is in the confinement phase and in the (u, x_4) coordinates we have the geometry of the cigar with the tip at $u = u_\Lambda$. The D4 branes are located along our D=4 geometry and are extended along x_4 coordinate. Let us emphasize that for the magnetic string we discuss the target space looks as $M_{10} \times CP(N-1)$ and involves the "internal" part.

Let us turn to our proposal for the magnetic string within the brane setup. Adopting the nonabelian string as the correct pattern we suggest that magnetic string at strong coupling regime is the probe D2 brane wrapped around S_1 parameterized by x_4 and its tension is therefore proportional to the effective radius $R(u)$. Due to the cigar geometry this wrapping is topologically unstable and the D2 brane shrinks to the tip where its tension vanishes. That is, one immediately reproduces the tensionless property of the magnetic string at zero temperature.

The next point concerns the worldsheet theory on D2 brane. We would like to see qualitatively the degrees of freedom corresponding to $CP(N-1)$ model as well as the correct θ -term in the worldsheet Lagrangian. The D2 brane supports U(1) gauge field on the worldvolume while the open D2-D4 strings provide the matter necessary for the gauge formulation of $CP(N-1)$ model. To trace the θ term let us consider the CS term on the D2 worldsheet

$$L_{CS} = \int d^3x \, C_1 \wedge F \quad (31)$$

where C_1 is the R-R one-form field. Taking into account that $\theta = \int dx_4 C_1$ we reproduce the standard θ -term in $CP(N-1)$ model $\theta \int d^2x F$.

The consideration of the finite N_C case is much more subtle. Let us consider finite N_C D4 brane wrapped around x_4 coordinate and a single D2 brane wrapped around x_4 as well. What could be the mechanism for the magnetic string condensation for finite N_C ? The natural conjecture is that the phenomena of dissolving of p-brane inside (p+2)-brane [22] takes place here. Indeed in our D2-D4 system we have proper brane dimensions and the tachyonic mode of D2-D4 open string could lead to the D2 brane condensation providing the additional magnetic field in four-dimensional theory via the CS term on the D4 worldsheet

$$L_{CS} = \int d^5x C_3 \wedge F \quad (32)$$

induced by D2 brane RR field.

However it is not clear how this D2-D4 tachyonic mode could disappear at the critical temperature and this point needs for the careful consideration. The possible stabilization mechanisms of p-brane inside (p+2)-brane which could be relevant in our context were discussed in [23] and are based on the account of RR fields in the bulk.

We are not able to describe the string condensation explicitly however one general comment is in order. It is known that the condensation of the electric charges results in confinement of the dual magnetic charges. The natural question concerns the nature of the dual object in our case and their possible "confinement". The magnetic string is assumed to be the source of the RR 2-form field $C_{\mu\nu}$ with non-vanishing curvature three-form $B_3 = dC_2$. The duality relation

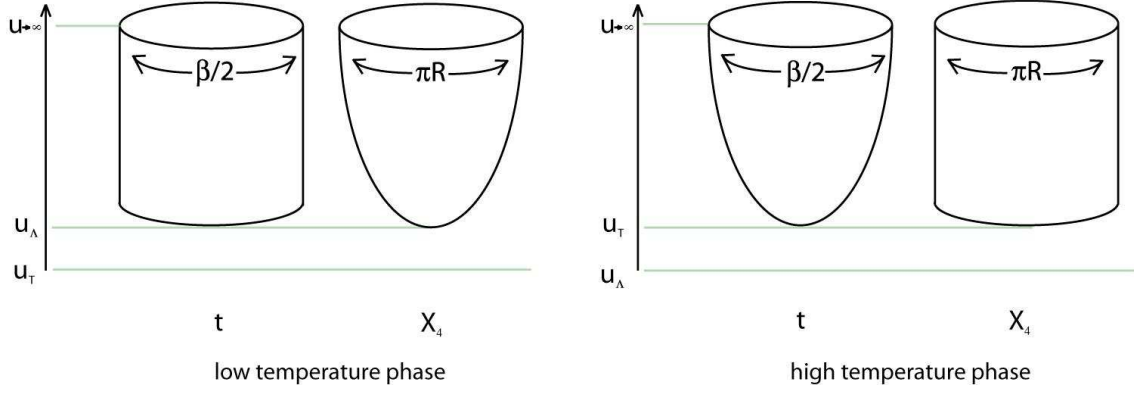
$$*B = da \quad (33)$$

imply that the dual field is pseudoscalar axion. Hence the dual objects which interact with the dual field are carriers of the topological charge and we could expect "confinement of the carriers of the topological charge" upon the magnetic string condensation.

3.5 Temperature dependence of the magnetic string

The crucial test of the proposal concerns the temperature dependence of the magnetic string. We have argued above that at zero and small temperatures the cigar geometry in (x_4, u) plane amounts to the vanishing tension of the magnetic string since the radius of the circle D2 brane wrapped around shrinks to zero. However the magnetic string becomes tensionful above the critical temperature T_c of the deconfinement phase transition. How the change of two regimes happens?

The key point is that in the temperature case there are two backgrounds with the asymptotic topology of $R^3 \times S_\tau^1 \times S^1 \times S^4$, where τ is the Wick rotated time coordinate $\tau = it$, $\tau \propto \tau + \beta$. One background corresponds to the analytic continuation of the previous one while the second corresponds to the interchange of τ and x_4 , that is the warped factor is attached to the τ coordinate and cigar geometry emerges in the (τ, u) plane instead of (x_4, u) plane which now exhibits the cylinder geometry, see Figure. It was shown in [19] by calculation of the free energies that above T_c the second background dominates.



Thus, above $T = T_c$ one gets the geometry of the cylinder in (x_4, u) and cigar in (τ, u) so that the wrapping around x_4 is topologically stable now and the magnetic string tension is proportional the cylinder radius. Moreover, by construction the D2 brane is wrapped around x_4 coordinate but the rest two coordinates of the D2 brane can fill the different dimensions. If both coordinates of the magnetic string are transverse to the time direction it does not feel the instability in the (τ, u) cigar geometry and behaves as S-string. On the other hand, if the magnetic string is wrapped around τ coordinate it is unstable in the cigar geometry and shrinks along the τ coordinate to zero. That is magnetic string extended in the time direction loses one physical dimension above T_c and, speaking somewhat loosely, looks as a "particle". One could say that vanishing tension below the critical temperature is "traded for" a lost dimension above the critical temperature.

Let us emphasize that we have discussed above the "space" tension corresponding to the linear density of the energy which jumps at the phase transition point. On the other hand the "internal" tension defining the $CP(N-1)$ part of the action of the magnetic string $T_{int} = 1/g^2$ goes smoothly at any temperature according to the asymptotic freedom. Remark that the internal part of the action follows from the open strings stretched between D2 brane and the rest of D4 branes.

Note that the discussion in this section is somewhat similar to the consideration in [21] of the role of the instantons in the similar geometry which are represented by Euclidean D0 branes wrapped around x_4 . In that case it was argued that single instanton is ill-defined below T_c because of D0 brane instability in the cigar geometry while above T_c it is well defined due to geometry of the cylinder. The change of the instanton role at the transition point corresponds to the change from the Witten-Veneziano to t'Hooft mechanisms of the solution to $U(1)$ problem.

4 Lattice data

4.1 Lattice strings at zero temperature

In this section we will provide a short guide to the literature on lattice measurements relevant to the theoretical issues discussed in this note.

Magnetic strings were introduced first in the context of the confinement studies, as

confining field configurations and are known mostly as ‘center vortices’, for review and references see [24]. In particular it was found that the vortices percolate in the vacuum, i.e. form an infinite cluster or a kind of condensate. Also, their total area of the vortices is in physical units,

$$(Area)_{total} \sim \Lambda_{QCD}^2 V_{total} \quad , \quad (34)$$

where V_{total} is the total volume of the lattice.

For confinement, the transverse size of the strings is not crucial and the strings were mostly thought of as ‘thick vortices’. The fact that they are actually thin, two-dimensional surfaces was discovered as a result of measurements of the distribution of nonabelian action associated with the vortices [25]. The action turned to be singular in the continuum limit,

$$(Action)_{lattice} \sim (Area)_{total}/a^2 \quad , \quad (35)$$

where a is the lattice spacing, $a \rightarrow 0$ in the continuum limit. Moreover, the nonabelian field living on the surface is aligned, or ‘trapped’ to the surface. It is these, thin strings which are relevant to our discussion. Moreover, the strings are closed in the vacuum state but can be open on an external ‘t Hooft line, for argumentation and references see [1]. Hence, the name of ‘magnetic strings’.

Note the physical string tension is not directly related to the lattice action but is to be rather calculated as a difference between lattice action and entropy factors, see, e.g., [26]. It is difficult to check such a cancelation directly. The fact that the physical tension for the lattice magnetic strings is vanishing in the confining phase follows from the very existence of an infinite, or percolating cluster of surfaces. Indeed, if the tension were not zero only finite clusters could be observed, by virtue of the uncertainty principle.

Lattice monopoles are identified as closed trajectories, or particles, for review see [27]. Their lattice algorithmic definition is independent of the definition of the surfaces, or strings. Nevertheless, the lattice simulations reveal that the monopole trajectories lie in fact on the surfaces [28, 25]. The nonabelian fields associated with the monopoles are also singular [29] and are aligned with the surfaces [25].

All the lattice data on the magnetic strings are obtained with the standard Wilson action and in fact refer mostly to pure Yang-Mills cases. There is no direct explanation of the data within the Yang-Mills theory itself. One can check, however, that the singular nonabelian fields are just of the type which is in no contradiction with the asymptotic freedom [30].

4.2 Lattice strings in the deconfinement phase

We also considered strings at non-zero temperature and here we will provide references to the lattice measurements at temperatures above the deconfinement phase transition.

The basic observation, made on the lattice [31, 24] is that at temperatures above the phase transition the strings become time-oriented. The four-dimensional infinite percolating cluster is dissolved and does not exist any longer. However, the percolation is not eliminated altogether. Namely, in three-dimensional slices the four-dimensional strings are projected to lines. In case of the magnetic strings, the properties of these lines depend crucially on whether one considers equal-time or equal-space-coordinate slices. In case of equal-time

slices the lines, which are intersections of the strings and of the 3d spaces, continue to percolate. In case of the equal-space-coordinate slices, there is no percolation at all.

Clearly, these lattice data are reproduced by the phenomenon of a ‘missed dimension’ discussed in detail in Section 3.5 in the brane language.

5 Discussion

During the last thirty years the derivation of the Mandestam’s qualitative explanation of the confinement via the dual Meissner effect was the main goal of the nonperturbative QCD studies. The recent lattice data suggests that probably the picture is to be modified and condensation of the tensionless magnetic strings takes place in QCD vacuum, instead of the condensation of the magnetic monopoles. If fact, there is no deep contradiction between two scenarios. Indeed the magnetic strings observed on the lattice support the monopoles at their worldsheets. In other words, condensation of the strings implicitly assumes the condensation the monopoles. The monopoles become, however, particles living on a string, or in 2d instead of ordinary particles living in 4d.

In this paper we conjecture that the strings observed on the lattice follow the pattern of the nonabelian strings with their rich worldsheet structure supporting monopole-antimonopole pairs. We have argued that this picture explains qualitatively quite a few effects observed on the lattice in pure Yang-Mills case. Moreover, it turns out that the interpretation of the magnetic strings as wrapped D2 branes fits perfectly with their properties.

What could we say about the wave function of the condensate? Since the lattice data indicate magnetic string condensate component in the vacuum the important point concerns the “phase of the condensate wave function”. In other terms the question is what is the remnant of the internal $CP(N-1)$ -type part of the action of the individual string in the condensate. We could speculate that the two-dimensional $CP(N-1)$ model on the string worldsheet is promoted to the four-dimensional $CP(N-1)$ sigma-model upon condensation. This four-dimensional model supports the BPS stringy solution which corresponds to the two-dimensional instanton solution of $CP(N-1)$ model lifted to four-dimensions. This presumably can be treated as the “confinement of instantons” in the dual picture since individual instanton does not exist from the topological reasons in 4d $CP(N-1)$ model.

In this paper we have focused on the pure Yang-Mills theory however the generalization to QCD with fundamental quarks is possible. In particular it is interesting to investigate the role of the magnetic strings in the chiral properties of the theory. In the brane setup we can add N_f flavor branes and analyze the dynamics in the Sakai-Sugimoto model [32] (see also [33] for the earlier papers). We have discussed magnetic strings that is wrapped D2 branes only. However there are other wrapped D4 and D6 branes in this setup which have an interesting interpretation. These issues shall be discussed elsewhere.

This project was started during the KITP at Santa Barbara program “QCD and String Theory” in the fall 2004 and we would like to thank the organizers for the very creative atmosphere. We would like to thank O. Andreev, M. Chernodub, A. DiGiacomo, F. Gubarev, M. Polikarpov, M. Shifman, A. Vainshtein and A. Yung for the discussions and comments.

We are grateful to M. Chernodub and A. Nakamura for providing us with the new lattice data prior the publication. The work of A.G. was partly supported by grants RFBR-06-02-17382 and INTAS-05-1000008-7865.

References

- [1] V. I. Zakharov, “ *Dual string from lattice Yang-Mills theory*”, AIP Conf. Proc. **756** (2005) 182, [ArXiv:hep-ph/050101];
“*From confining fields on the lattice to higher dimensions in the continuum*,” Brazilian J. Phys. **37**, (2007) 165. [arXiv:hep-ph/0612342].
- [2] M. N. Chernodub and V. I. Zakharov, “Magnetic component of Yang-Mills plasma,” Phys. Rev. Lett. **98**, 082002 (2007) [arXiv:hep-ph/0611228].
M. N. Chernodub and V. I. Zakharov, “Magnetic strings as part of Yang-Mills plasma,” arXiv:hep-ph/0702245.
- [3] M.N. Chernodub, A. Nakamura, presented at the workshop “The Many Faces of Quantum Fields”, Leiden, The Netherlands, April 2007, and to be published.
- [4] A. Hanany and D. Tong, “Vortices, instantons and branes,” JHEP **0307**, 037 (2003) [arXiv:hep-th/0306150].
- [5] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi and A. Yung, “Nonabelian superconductors: Vortices and confinement in $N = 2$ SQCD,” Nucl. Phys. B **673**, 187 (2003) [arXiv:hep-th/0307287].
- [6] A. Gorsky, M. Shifman and A. Yung, “Non-Abelian Meissner effect in Yang-Mills theories at weak coupling,” Phys. Rev. D **71**, 045010 (2005) [arXiv:hep-th/0412082].
- [7] D. Tong, “TASI lectures on solitons,” arXiv:hep-th/0509216.
- [8] M. Shifman and A. Yung, “Supersymmetric Solitons and How They Help Us Understand Non-Abelian Gauge Theories,” arXiv:hep-th/0703267.
- [9] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Solitons in the Higgs phase: The moduli matrix approach,” J. Phys. A **39**, R315 (2006) [arXiv:hep-th/0602170].
- [10] D. Tong, “Monopoles in the Higgs phase,” Phys. Rev. D **69**, 065003 (2004) [arXiv:hep-th/0307302].
- [11] M. Shifman and A. Yung, “Non-Abelian string junctions as confined monopoles,” Phys. Rev. D **70**, 045004 (2004) [arXiv:hep-th/0403149].
- [12] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, “Two-Dimensional Sigma Models: Modeling Nonperturbative Effects Of Quantum Chromodynamics,” Phys. Rept. **116**, 103 (1984)

- [13] E. Witten, “Instantons, The Quark Model, And The $1/N$ Expansion,” Nucl. Phys. B **149**, 285 (1979).
- [14] M. Shifman and A. Yung, “Confinement in $N=1$ SQCD: One Step Beyond Seiberg’s Duality,” arXiv:0705.3811 [hep-th].
- [15] M. Eto, K. Hashimoto and S. Terashima, “QCD String as Vortex String in Seiberg-Dual Theory,” arXiv:0706.2005 [hep-th].
- [16] A. Gorsky, M. Shifman and A. Yung, “Nonabelian strings and axion,” Phys. Rev. D **73**, 125011 (2006) [arXiv:hep-th/0601131].
- [17] E. Witten, “Anti-de Sitter space, thermal phase transition, and confinement in gauge theories,” Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131].
- [18] N. Dorey, “The BPS spectra of two-dimensional supersymmetric gauge theories with twisted mass terms,” JHEP **9811**, 005 (1998) [arXiv:hep-th/9806056].
N. Dorey, T. J. Hollowood and D. Tong, “The BPS spectra of gauge theories in two and four dimensions,” JHEP **9905**, 006 (1999) [arXiv:hep-th/9902134].
- [19] O. Aharony, J. Sonnenschein and S. Yankielowicz, “A holographic model of deconfinement and chiral symmetry restoration,” Annals Phys. **322**, 1420 (2007) [arXiv:hep-th/0604161].
- [20] A. Gorsky, M. Shifman and A. Yung, “ $N = 1$ supersymmetric quantum chromodynamics: How confined non-Abelian monopoles emerge from quark condensation,” Phys. Rev. D **75**, 065032 (2007) [arXiv:hep-th/0701040].
- [21] O. Bergman and G. Lifschytz, “Holographic $U(1)_A$ and string creation,” JHEP **0704**, 043 (2007) [arXiv:hep-th/0612289].
- [22] E. Gava, K. S. Narain and M. H. Sarmadi, “On the bound states of p- and (p+2)-branes,” Nucl. Phys. B **504**, 214 (1997) [arXiv:hep-th/9704006].
- [23] E. J. Copeland, R. C. Myers and J. Polchinski, “Cosmic F- and D-strings,” JHEP **0406**, 013 (2004) [arXiv:hep-th/0312067].
L. Leblond and S. H. H. Tye, “Stability of D1-strings inside a D3-brane,” JHEP **0403**, 055 (2004) [arXiv:hep-th/0402072].
- [24] J. Greensite, “*The Confinement problem in lattice gauge theory*”, Progr. Part. Nucl. Phys. **51** (2003) 1, [arXiv:hep-lat/0301023].
- [25] F. V. Gubarev, A. V. Kovalenko, M. I. Polikarpov, S. N. Syritsyn, V. I. Zakharov, “*Fine tuned vortices in lattice $SU(2)$ gluodynamics*”, Phys. Lett. **B574** (2003) 136, [arXiv:hep-lat/0212003].
- [26] A.M. Polyakov, “*Gauge Fields and Strings*”, Harvard Academic Publishers, (1987);
J. Ambjorn, “*Quantization of geometry*”, [arXiv:hep-th/9411179].

- [27] M.N.Chernodub, F.V.Gubarev, M.I.Polikarpov, A.I.Veselov, “*Monopoles in the Abelian Projection of Gluodynamics*”, Prog. Theor. Phys. Suppl. **131** (1998) 309, [arXiv:hep-lat/9802036];
A. Di Giacomo, “*Monopole condensation and color confinement*”, Prog. Theor. Phys. Suppl. **131** (1998) 161, [arXiv:hep-lat/9802008];
T. Suzuki, “*Low-energy effective theories from QCD*”, Prog. Theor. Phys. Suppl. **131** (1998) 633.
- [28] J. Ambjorn, J. Giedt, J. Greensite, S. Olejnik, “*Vortex structure versus monopole dominance in Abelian projected gauge theory*”, JHEP **0002** (2000) 033, [arXiv:hep-lat/9907021].
- [29] V.G. Bornyakov, et al., “*Anatomy of the lattice magnetic monopoles*”, Phys. Lett. **B537** (2002) 291, [arXiv:hep-lat/0103032].
- [30] V.I. Zakharov, “*Nonperturbative match of ultraviolet renormalon*”, [arXiv:hep-ph/0309178].
- [31] M. Engelhardt, K. Langfeld, H. Reinhardt, O. Tennert, “*Deconfinement in $SU(2)$ Yang-Mills theory as a center vortex percolation transition*”, Phys. Rev. **D61** (2000) 054504, [arXiv:hep-lat/9904004].
- [32] T. Sakai and S. Sugimoto, “Low energy hadron physics in holographic QCD,” Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141].
T. Sakai and S. Sugimoto, “More on a holographic dual of QCD,” Prog. Theor. Phys. **114**, 1083 (2006) [arXiv:hep-th/0507073].
- [33] A. Karch and E. Katz, “Adding flavor to AdS/CFT,” JHEP **0206**, 043 (2002) [arXiv:hep-th/0205236].
M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, “Towards a holographic dual of large- $N(c)$ QCD,” JHEP **0405**, 041 (2004) [arXiv:hep-th/0311270].